GAME THEORY

Pay Off Matrix

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1. **What is a Pay off Matrix?**

Ans : In a two person zero or constant sum game, the resulting gain can be represented in the form of a matrix which is called Pay-off Matrix or Gain matrix. In other words payoff matrix is a table in which strategies of one player are listed in rows and those of the other player in columns and the cells show payoffs to each player such that the payoff of the row player is listed first.

A payoff matrix is an important tool in game theory because it summarizes the necessary information and helps us determine whether a dominant strategy or a Optimal Mixed strategy or a Nash equilibrium exist. In Economics Pay off matrices are widely used while represent ting a game of Duopoly or Oligopoly model.

Description of a Pay off Matrix of a two person game.

Let us consider a two person game with players A and B. Player A has *m* possible courses of action and player B has *n* possible courses of action. The game can be described by means of a pair of Pay off matrices in the following way.

a) The row designation for each Matrix are the courses of action available to player A.
b) The column designation for each Matrix are the courses of action available to player B.
c) The cell entries in the Pay off matrix of A are the payments to A at the end of a Play. The cell entry a_{ij} is the payment to A in A's Pay-off matrix when A chooses the *'i'th* course of action and B chooses the '*j'th* course of action.
d) The cell entries in the Pay off matrix of B are the payments to B at the end of a Play. The cell entry b_{ij} is the payment to B in B's Pay-off matrix when A chooses the *'i'th* course of action and B chooses the '*j'th* course of action.
e) In a two person zero sum game, the cell entries (b_{ij}) in B's Pay off matrix are the negatives of the corresponding cell entries (a_{ij}) in A's Pay off matrix so that $a_{ij} + b_{ij} = 0$.
f) In a two person constant sum game, the cell entries (b_{ij}) in B's Pay off matrix are related of the corresponding cell entries (a_{ij}) in A's Pay off matrix in such a way that.
   $a_{ij} + b_{ij} = k$. Where k is a constant.

The two Pay off Matrices are as follows.

For a Two Person Zero Sum Game

A's Pay off Matrix

$$
\begin{array}{cc|ccccccc|}
\multicolumn{9}{c}{B} \\
 & & 1 & 2 & 3 & \cdots & j & \cdots & n \\ \hline
 & 1 & a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\
 & 2 & a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\
 & 3 & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
A & . & . & . & . & \cdots & . & \cdots & . \\
 & i & a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} & \cdots & a_{in} \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & m & a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn} \\ \hline
\end{array}
$$

B's Pay off Matrix

$$
\begin{array}{cc|ccccccc|}
\multicolumn{9}{c}{B} \\
 & & 1 & 2 & 3 & \cdots & j & \cdots & n \\ \hline
 & 1 & -a_{11} & -a_{12} & -a_{13} & \cdots & -a_{1j} & \cdots & -a_{1n} \\
 & 2 & -a_{21} & -a_{22} & -a_{23} & \cdots & -a_{2j} & \cdots & -a_{2n} \\
 & 3 & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
A & . & . & . & . & \cdots & . & \cdots & . \\
 & i & -a_{i1} & -a_{i2} & -a_{i3} & \cdots & -a_{ij} & \cdots & -a_{in} \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & . & . & . & . & \cdots & . & \cdots & . \\
 & m & -a_{m1} & -a_{m2} & -a_{m3} & \cdots & -a_{mj} & \cdots & -a_{mn} \\ \hline
\end{array}
$$

For a Two Person Constant Sum game

A's Pay off Matrix

		B						
		1	2	3	. . . .	j	. . . . . .	n
	1	a_{11}	a_{12}	a_{13}	. . . .	a_{1j}	. . . . . .	a_{1n}
	2	a_{21}	a_{22}	a_{23}	. . . .	a_{2j}	. . . . . .	a_{2n}
	3	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
A	.	.	.	.	. . . .	.	. . . . . .	.
	i	a_{i1}	a_{i2}	a_{i3}	. . . .	a_{ij}	. . . . . .	a_{in}
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	m	a_{m1}	a_{m2}	a_{m3}	. . . .	a_{mj}	. . . . . .	a_{mn}

B's Pay off Matrix

		B						
		1	2	3	. . . .	j	. . . . . .	n
	1	$k-a_{11}$	$k-a_{12}$	$k-a_{13}$	. . . .	$k-a_{1j}$	. . . . . .	$k-a_{1n}$
	2	$k-a_{21}$	$k-a_{22}$	$k-a_{23}$	. . . .	$k-a_{2j}$	. . . . . .	$k-a_{2n}$
	3	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
A	.	.	.	.	. . . .	.	. . . . . .	.
	i	$k-a_{i1}$	$k-a_{i2}$	$k-a_{i3}$	. . . .	$k-a_{ij}$	. . . . . .	$k-a_{in}$
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	.	.	.	.	. . . .	.	. . . . . .	.
	m	$k-a_{m1}$	$k-a_{m2}$	$k-a_{m3}$	. . . .	$k-a_{mj}$	. . . . . .	$k-a_{mn}$