GAME THEORY

Solution of a game Without a Saddle point.

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1. **Explain how a rectangular game without a saddle point is solved.**

   Ans: If a Rectangular game does not have a Saddle point, the two players cannot use Pure Strategies i.e. Maximin and Minimax criterion of optimality. Then the best strategies are Mixed strategies. The two players, instead of selecting one strategy may play their plays according to some predetermined set which consists of probabilities corresponding to each of their pure strategies.

   Properties of Optimal Mixed strategies:

   1. If one of the players adheres to his optimal mixed strategy and the other deviates from his optimal strategy, then the deviating player cannot increase his pay off. In other words by deviating from his Optimal Mixed strategy a player can only decrease his yield.
   2. If one of the players adheres to his optimal mixed strategy then the value of the game does not change if the opponent uses his/her supporting strategies directly or in mix.
   3. If a constant c is added to each element of the pay off matrix, the optimal strategies remain unchanged while the value of the game increases by c.
   4. If every element of the pay off matrix is multiplied by a constant c , then the optimal strategies remain unchanged and the value of the game increases by c times.

   SOLUTION OF A 2X2 GAME WITHOUT A SADDLE POINT

   A rectangular game without a saddle point cannot be solved using pure strategies or with minimax and Maximin criterion of optimality. So, in this case the best strategies are the Mixed strategies. In this strategy the probability with which each action should be selected is calculated.

   The following theorem is followed to find the solution of a two person 2X2 pay off matrix without a saddle point.

   For any two person game where the optimal strategies are not pure, but mixed strategies and for which A’s Pay off Matrix is

$$\begin{array}{cc|cc|} & & \multicolumn{2}{c}{B} \\ & & \text{I} & \text{II} \\ & & y_1 & y_2 \\ \hline & \text{I} & a_{11} & a_{12} \\ A & x_1 & & \\ & \text{II} & a_{21} & a_{22} \\ & x_2 & & \\ \hline \end{array}$$

The optimal mixed strategies of player A will be (x_1, x_2) and that of player B will be (y_1, y_2). The values of x_1, x_2, y_1, y_3 are given by –

$$x_1 = \frac{(a_{22} - a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$x_2 = \frac{(a_{11} - a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$y_1 = \frac{(a_{22} - a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$y_2 = \frac{(a_{11} - a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

The value of the game is given by

$$v = \frac{(a_{11}a_{22} - a_{12}a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Q: What is Dominance Property? What are the rules of dominance?

Ans : When for each pure strategy of the opponent's the payoff associated with One strategy is no better than another strategy of the player then the first strategy is said to be dominated by the second strategy and the player will never use that strategy. Since the dominated pure strategy will never be a part of the optimal mixed strategy , it can be omitted from the pay off Matrix. In other words If in a rectangular game one or more of the pure strategies of a player are inferior to at least one of the remaining strategies ,the later is said to dominate the other strategies and the player will never use the dominated strategies . Hence these dominated strategies can be deleted from the pay off matrix and thus the size of the pay off matrix can be reduced.

Rules of Dominance:

The following rules or properties of dominance are used to reduce the size of the pay – off matrix.

1. If all the elements in a row (i^{th} row)of a pay off matrix are less than or equal to the corresponding elements of another row (j^{th} row) then the i^{th} row is said to dominated by j^{th} row and can be deleted from the pay off matrix as player A will never use that dominated strategy.
2. If all the elements in a column (p^{th} column)of a pay off matrix are more than or equal to the corresponding elements of another column (q^{th} column) then the p^{th} column is said to dominated by q^{th} column and can be deleted from the pay off matrix as player B will never use that dominated strategy.
3. If all the elements in a row (i^{th} row)of a pay off matrix are less than or equal to the average of corresponding elements of two or more other rows then the i^{th} row is said to dominated by *the other* rows and can be deleted from the pay off matrix as player A will never use that dominated strategy.
4. If all the elements in a column (p^{th} column)of a pay off matrix are more than or equal to the average of corresponding elements of two or more other columns then the p^{th} column is said to dominated by *the other* columns and can be deleted from the pay off matrix as player B will never use that dominated strategy.

   By using the rules of dominance the size of the pay off matrix is reduced to 2X2 matrix, and then it is solved by the Optimal Mixed strategies.